Measuring the Compactness of Legislative Districts

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Measuring the Compactness Of Legislative Districts

About half of the states currently impose some form of compactness standard on their districting plans. Yet there is no commonly accepted definition of how compactness is to be measured. This paper surveys some of the principal compactness measures gleaned from the literature in political science, geography, law, and mathematics. Although some of these measures are better than others, all are defective in some crucial respect; that is, they fail to give satisfactory results on certain types of geographical configurations. The conclusion is that "compactness" is not a sufficiently precise concept to be used as a legal standard for districting plans.

According to Webster (1961), a compact figure is one that is "homogeneous and located within a limited definite space without straggling or rambling over a wide area." While this definition may appeal to our intuition, it does not provide a rigorous or precise standard that can be used to determine whether a districting plan is or is not compact. The question is whether any such definition of compactness exists. In this paper we shall survey the principal measures that have been suggested in the literature. In particular, we shall examine the methods of Roeck, Schwartzberg, Taylor, and Boyce-Clark, as well as other criteria such as the moment of inertia, the perimeter test, and length-to-width ratios. We shall then illustrate, by example, how these methods work and whether they give intuitively reasonable results.

A general distinction must be made between the compactness of a district and the overall compactness of a districting plan. Most of the measures surveyed here are defined in terms of single districts. An entire districting plan can be evaluated either by testing each district individually for compactness or by summing the measure over all of the component districts in the plan. The latter process has the obvious drawback that it really only measures the average compactness of a plan; it may allow individual districts that are of very doubtful shape to slip by even though the plan as a whole passes muster. Hence it seems better to insist that every district taken by itself be "compact." Unfortunately, this is not always possible. If the state itself is geographically "straggling and rambling over a wide area" and if the number of districts is small, then some districts, taken by themselves, may necessarily be noncompact by any reasonable measure.

Since truly compact districts of nearly equal population may be impossible to achieve, the best that can be hoped for is a standard that measures the relative compactness of different plans. One would choose the most compact plan from among those that meet other criteria, such as equality of populations. Unfortunately, even then there appears to be no completely satisfactory criterion for telling whether one plan is more compact than another. The problem seems to be inherent in the notion of compactness itself. We conclude that compactness is not a very useful or operational criterion for judging whether a districting plan is fair.

Eight Measures of Compactness

In this section we shall examine the measures most commonly used or cited in the literature. In each case we first define the criterion, then see whether it gives intuitively reasonable and reliable results.

The Visual Test

The simplest of all tests is to use the eye and intuition. But appearances can be deceiving and intuition may fail in even the simplest cases. Consider Figures 1-4, each having about the same area: which is most compact?

The Roeck Test

Find the smallest circle containing the district and take the ratio of the district's area to that of the circle. This ratio is always between 0 and 1; the closer it is to 1, the more compact is the district (Roeck, 1961).

According to the Roeck measure, the star (Figure 3) is the most compact, with ratio .55; the triangle (Figure 2), with ratio .32, is the least compact. This result does not seem very convincing. In general, the problem with the Roeck measure is that it can give a high rating (near 1) to an arbitrarily misshapen district so long as it meanders around within a confined area. For example, the coiled snake of Figure 5 is, by the Roeck measure, more compact than a simple square district. On the other hand, it can give a low rating to perfectly reasonable districts, such as the triangle in Figure 2.

FIGURE 1
Elbow

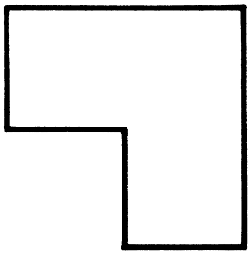


FIGURE 2
Triangle

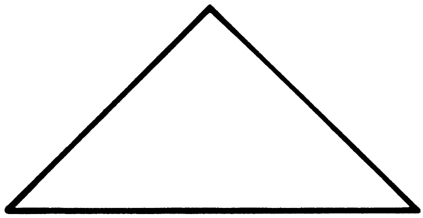


FIGURE 3
Star

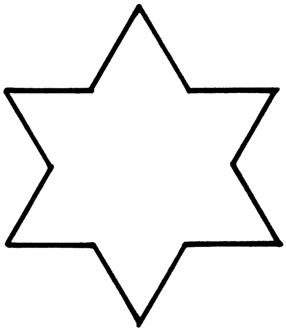


FIGURE 4
Apple

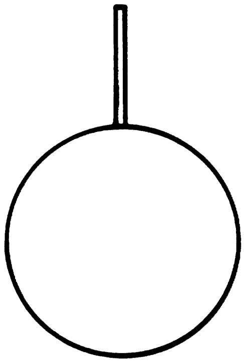


FIGURE 5
Serpent

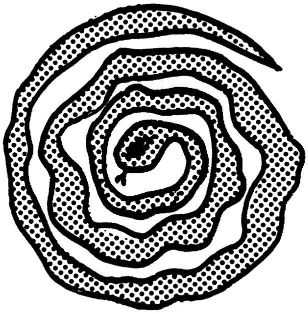
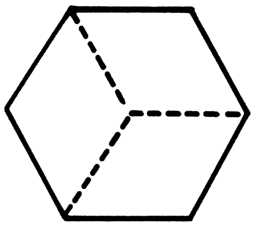


FIGURE 6
Hexagon



The idea has sometimes been advanced that a district with Roeck measure less than .4 should be deemed noncompact and rejected (Hacker, 1963). Such a threshold may be impossible to satisfy by any districting plan. For example, it would be impossible to divide a hexagon into three equal-sized districts as in Figure 6 without violating the .4 threshold. Therefore, this is obviously not a reasonable standard, nor does it have any logical foundation in theory. In his original article, Roeck himself cautioned against the use of thresholds.

Perhaps the logical concluding step in this discussion should be recommendations for statutory provisions requiring each district to meet at least a minimum standard of compactness. This is the approach taken with regard to population equality; a maximum relative deviation is proposed—sometimes 10, sometimes 15, sometimes 20%. Such a step is not desirable for compactness. In states such as Oklahoma or Maryland, with sparsely populated "panhandles," some districts must inevitably lack compactness. . . . Furthermore, other apportionment standards may well take precedence, with population equality heading the list (1961, pp. 73-74).

The Schwartzberg Test

Construct the adjusted perimeter of the district by connecting by straight lines those points on the district boundary where three or more constituent units (i.e., census tracts) from any district meet. Divide the length of the adjusted perimeter by the perimeter of a circle with area equal to that of the district[1] (Schwartzberg, 1966).

By the Schwartzberg measure, the elbow is the most compact of the first four figures (with a measure of 1.3), whereas the star is the least compact (with a measure of 1.5).

The Schwartzberg measure is defective in that it places too much emphasis on the perimeter and not enough on the overall shape. For example, the need to give due regard to boundaries of political subunits may force the perimeter to be quite long even though the district itself is reasonably compact. Figure 8 shows a nearly circular district whose boundary is irregular because it is composed of square subunits. By the Schwartzberg measure this figure is actually less compact than the dog-leg depicted in Figure 7.

It has sometimes been suggested that districts with a Schwartzberg measure greater than 1.67 should be deemed noncompact and rejected (Wiser v. Hughes, 1982). By this standard the nearly circular district of Figure 8 would be rejected and the dog-leg accepted, so it hardly seems to be a satisfactory test. Schwartzberg, while suggesting a 1.67 cutoff as "reasonable" for the case of congressional districts in North Carolina (89th Congress), admits that "any maximum figure chosen will be arbitrary" (Schwartzberg, 1966, p. 446).

FIGURE 7
Dog-leg

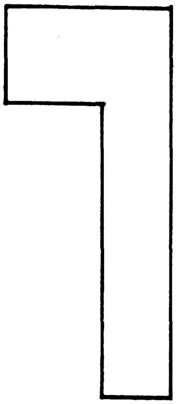
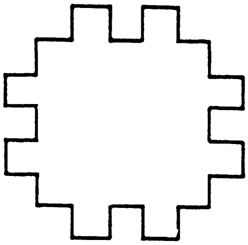


FIGURE 8
Puzzle Piece



Length-width Test

Find a rectangle enclosing the district and touching it on all four sides, such that the ratio of length to width is a maximum. The closer the ratio is to 1, the more compact is the district.

Several variations of this idea appear in the literature (see U.S. Congress, 1959, statement of Joseph Kellenbach; Harris, 1964; Papayanopoulos, 1973). It is not a reliable measure of compactness, since it can give a near-perfect rating to devious, unnatural figures such as the "salamander" shown in Figure 9.

Another absurdity is that by this measure the sawtooth division of the square shown in (Figure 10) is preferable to a straight cut. Some have suggested (Wiser v. Hughes, 1982) that districts with a length-to-width ratio greater than 1.75 should be deemed noncompact. Such a standard is arbitrary and leads to nonsensical results. For example, it is

FIGURE 9
Salamander

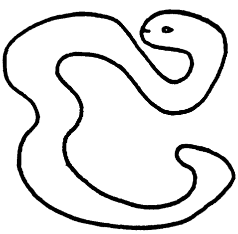


FIGURE 10
Sawtooth

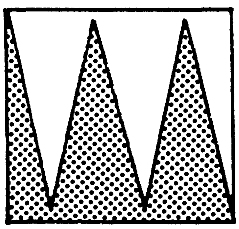


FIGURE 11
Squeezed Square

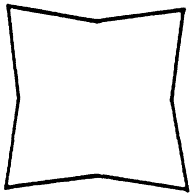


FIGURE 12
Narrow Strip

not possible to cut a square into two equal-sized convex districts without both of them failing the test.

A variation of this measure is to consider the maximum difference between the length and the width of a circumscribing rectangle (Papayanopoulos, 1973). This is also inappropriate as a measure of gerrymandering, since it means that large rural districts are necessarily less compact than small urban districts having the same shape, which implicitly allows greater latitude for gerrymandering in urban than in rural districts.

Taylor's Test

Construct the adjusted perimeter of the district by connecting by straight lines those points on the district boundary where three or more constituent subunits from any district meet. At each such point the angle formed is "reflexive" if it bends away from the district and "nonreflexive" otherwise. Subtract the number of reflexive from the number of nonreflexive angles and divide by the total number of angles. The resulting number is always between 0 and 1; the closer to 1, the more compact the district (Taylor, 1973).

By this measure the triangle is perfectly compact, while the star is perfectly noncompact. It is a defective measure because it can give a 0 rating to districts that are very nearly square (Figure 11) and a perfect 1 rating to extremely elongated districts (Figure 12).

The Moment of Inertia Test

Locate the geographical center c_i of each census tract i in the district. Select an arbitrary point x and calculate the square of the distance from x to c_i, multiplied by the population of tract i. The sum of these numbers is the district's moment of inertia about the point x. That point which gives the

minimum moment of inertia is the center of gravity of the district. The smaller the moment of inertia about the center of gravity the more compact is the district (Weaver and Hess, 1963).

The defect of this measure is that it gives good ratings to misshapen districts so long as they meander within a confined area, as in Figure 5. Furthermore, the test is not only a measure of compactness, it is a measure of size as well. For example, a large rural district will necessarily have a worse measure of compactness than a small urban district of the same shape. Thus the test discriminates against rural districts.

The Boyce-Clark Test

Determine the center of gravity of the district and measure the distance from the center to the outside edges of the district along equally-spaced radial lines. Compute the percentage (±) by which each radial distance differs from the average radial distance, and find the average of the percentage deviations over all radials. The closer the result is to 0, the more compact is the district (Boyce and Clark, 1964).

The defect of the Boyce-Clark measure is that it gives excellent ratings to devious districts like the snake (Figure 5) and the salamander (Figure 9).

The Perimeter Test

Find the sum of the perimeters of all the districts. The shorter the total perimeter, the more compact is the districting plan.

This measure differs from most of the others in that it applies to a whole plan rather than to a single district. It has been advocated by various authors, including Dixon (1968), Tyler and Wells (1971), and Adams (1977), as a criterion to be minimized subject to meeting population equality and other fairness objectives.

As a practical matter, determining what districting plan actually does have the shortest possible perimeter may involve extremely difficult computations. Moreover, the measure is theoretically defective. First, long perimeters do not necessarily imply noncompact districts, as shown by Figure 8. Second, use of the measure to compare alternative plans might tend to promote misshapen or gerrymandered urban districts at the expense of small improvements in the shape of rural districts, whose

FIGURE 13
Embryonic Gerrymander

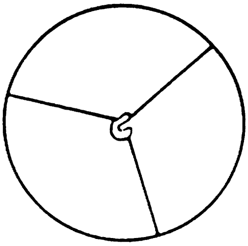
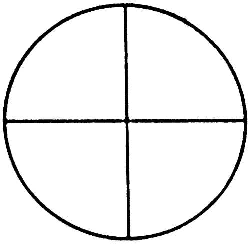


FIGURE 14
Quadrants



boundaries are naturally much longer. Figures 13 and 14 show two ways of dividing into four districts a circular state having higher population density near the center than on the boundary. The apparently gerrymandered plan of Figure 13 does better by the perimeter test than the evidently fairer plan of Figure 14.

Desirable Properties of Compactness Measures

There appears to be no mathematical measure of compactness in the literature that gives a reliable test of whether a districting plan is or is not reasonably compact. Every one of the measures known to this author is defective in some respect. To illustrate this point we have relied on artfully concocted examples, some of which (it might be said) are improbable and even verge on the fanciful. In fact these examples are not all that special. They simply highlight certain general principles that, taken together, are incompatible with every one of the measures touted in the literature.

We may summarize these principles as follows:

First, there is no single threshold or cut-off value that establishes when a district or a districting plan fails to be compact. A measure can only indicate when one plan is more compact than another.

Second, a compactness measure should apply to the whole districting plan as well as to individual districts. Measures that apply only to single districts fail to take into account that if a small number of districts is required in a state with irregular boundaries—for example, Maryland—then some of the districts will necessarily be very noncompact in appearance. Contrariwise, measures that apply only to the whole plan may allow a few grossly gerrymandered districts to slip through.

Third, a measure should treat predetermined political units or census tracts as indivisible building blocks whose shape is irrelevant to the measure. The "moment of inertia" measure is desirable from this standpoint, since it is based on the distances of different tracts from a reference point. The Schwartzberg concept of using the "adjusted" rather than the true perimeter also partially satisfies this criterion.

Fourth, a measure should not discriminate between large rural and small urban districts; it should measure only the shape, not the size of the district. The Weaver-Hess and Schwartzberg tests can be faulted on these grounds, whereas pure measures of shape, such as the Roeck, length-to-width, Boyce-Clark, and Taylor tests, are superior. Figure 13 illustrates why a measure based on absolute distance allows greater gerrymandering of urban than rural districts.

Fifth, a measure should be conceptually simple and require only easily collected, readily verifiable data. Data requiring elaborate surveys are open to attack on the grounds of inaccuracy and inconvenience. Measures requiring circumscribing geometrical figures or adjusted perimeters are also problematical because of the potential for error.

Conclusion

The chief rationale for invoking a compactness standard is to restrict the latitude for manipulating the political composition of districts. Since the courts currently regard equality of population as the primary criterion for judging the fairness of plans, the most that could be demanded of a compactness measure would be to apply it only when a more compact plan is attainable that meets a given standard of equality in populations (such as a maximum of 1% deviation from the ideal population). The truth of the matter is, however, that compactness is such a hazy and ill-defined concept that it seems impossible to apply it, in any rigorous sense, to matters of law.

There is also a deeper issue here. In recent years the courts have been tempted to rely increasingly on mathematical formulae to assess the fairness of districting plans. Ever-finer definitions of what constitutes population equality have been invoked. And there have been many attempts to formalize the notion of compactness in terms of a mathematical standard, most of them ill-conceived. This reliance on formulas has the semblance, but not the substance, of justice. It opens the door to more subtle types of gerrymandering, in which high-speed computers manipulate data bases in order to create plans that meet superficial mathematical criteria of equality and compactness, while being grossly gerrymandered in the political sense. We would argue, therefore, that compactness should either be abandoned as a standard altogether, or left in the domain of the dictionary definition, to be interpreted by the courts in the light of circumstances.

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NOTES

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1. A variation suggested by Schwartzberg is to use a circle with area equal to that enclosed by the adjusted perimeter, instead of the area of the district itself. Another variation, cited by Papayanopoulos (1973) is to take the ratio of the district's perimeter to its area. This variation has the unfortunate property that a large rural district will be necessarily less compact than a small urban district of exactly the same shape. Thus it implicitly allows greater latitude for gerrymandering in urban than in rural districts.

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