# Appendix — a table of counts for plane mechanisms

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*Abstract* — **We give a table of counts of the isomorphism equivalence classes of "plane mechanisms" with $L$ links and $\kappa$ degrees of freedom for $1 \le L \le 14$ and $0 \le \kappa \le 8$. These counts were based on a graphical characterization given by Warren D. Smith in the accompanying paper, and were generated by computer. We give a brief description of how the computer program works.**

B.McKay [3] discovered methods for isomorph-free generation of combinatorial objects. Similar methods were discovered independently by Henry Cejtin. Both used them to count various kinds of graphs. At Smith's request, Cejtin specialized a version of his program to enumerate 2D mechanisms. Here's a rough explanation of how Cejtin's program works in our particular case. $(V + 1)$-vertex graphs are generated inductively from $V$-vertex graphs. Throughout, only "canonical" forms of graphs, e.g. those whose adjacency matrices are lexically minimal under all permutations of the vertices, are kept; non-canonical graphs are discarded. This eliminates redundant isomorphs. It may be implemented to run surprisingly quickly by viewing the generation processes (both of graphs and of permutations) as trees whose non-minimal branches are pruned as soon as they are detected. For an *average* graph, it is extremely easy to prune almost everything away; although especially symmetric graphs could exist which cause the canonicity tester to take a long time, this apparently does not matter much because such graphs are rare. Also, Cejtin maintains a list of all pairs $(a, b)$ of vertices whose transposition is a graph automorphism, and his permutation generator does not generate permutations equivalent via such transpositions to a lexically smaller permutation. Graphs disobeying any subgraph flexibility condition are also discarded.

Finally, the graphs were bucketed according to how many vertices and degrees of freedom they had, and what their edge-connectivity was (a linear-time 2-edge-connectivity tester is outlined in [2] [1]), and the counts in each bucket were put in table 1.

## 1 The table

These counts are PRELIMINARY; later we intend to do a longer run (at least up through 13 vertices) and also count disconnected, 1-edge-connected, and ($\ge$ 2)-edge-connected mechanism describing graphs (3 connectivity categories instead of the present 2).

The counts with $\kappa = 0$ refer to "primitive rigid frameworks" (frameworks that are rigid, but no subframework is rigid). The number of degrees of freedom $\kappa$ is calculated assuming the lengths are generic and regarding one of the links as immovable.

WELL???? ALSO WOULD BE NICE TO PROVIDE THE SMALLER GRAPHS THEMSELVES IN ELECTRONIC FORM ON A WEB PAGE...

Right now, here are the counts for all graphs with 11 or fewer vertices, plus some more counts up through 14. Each table entry $a/b$ means there are $a$ non-connected and $b$ connected graphs meeting the conditions. $L$ is the number of links (corresponds to the number of vertices in the graph) and $\kappa$ is the number of degrees of freedom. Counts for graphs with $\kappa \ge 9$ are omitted.

## References

[1] Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest: Introduction to algorithms, MIT Press 1990.

[2] Shimon Even: Graph algorithms, Computer Science Press 1979.

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| $L$ | $\kappa = 0$ | $\kappa = 1$ | $\kappa = 2$ | $\kappa = 3$ | $\kappa = 4$ | $\kappa = 5$ | $\kappa = 6$ | $\kappa = 7$ | $\kappa = 8$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0/1 | | | | | | | | |
| 2 | | 0/1 | | 1/0 | | | | | |
| 3 | 0/1 | | 0/1 | | 1/0 | | 1/0 | | |
| 4 | | 0/1 | | 0/2 | | 2/0 | | 1/0 | |
| 5 | 0/1 | | 0/2 | | 1/3 | | 3/0 | | 2/0 |
| 6 | | 0/2 | | 0/6 | | 3/6 | | 7/0 | |
| 7 | 0/3 | | 0/9 | | 2/15 | | 9/11 | | 14/0 |
| 8 | | 0/16 | | 0/44 | | 12/45 | | 27/23 | |
| 9 | 0/28 | | 0/111 | | 16/173 | | 57/123 | | 85/47 |
| 10 | | 0/230 | | 0/728 | | 129/683 | | 255/358 | |
| 11 | 0/562 | | 0/2673 | | 230/3853 | | 868/2520 | | 1080/1013 |
| 12 | | 0/6856 | | ?? | | ?? | | ?? | |
| 13 | ?? | | 0/105082 | | ?? | | ?? | | ?? |
| 14 | | 0/318162 | | ?? | | ?? | | ?? | |

Table 1: Unconnected/connected counts of isomorphism equivalence classes of planar mechanisms with $L$ rigid links and $\kappa$ degrees of freedom.

[3] B.D.McKay: Isomorph free exhaustive generation, J.Algorithms 26,2 (Feb. 1998) 306-324.