Homework problem set #2 for "Computing and the laws of physics."

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Due: Tues Oct 23)

Do only 2 of the first 3 problems.

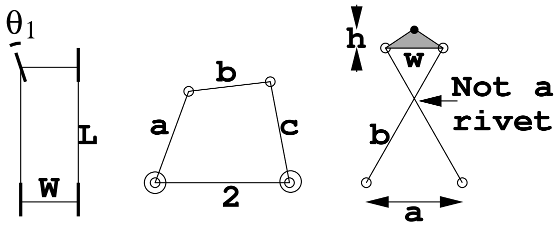

Figure 1: Scenarios in problems 1,2,3

1. A car has 4 thin wheels (when projected onto the plane of the road, these become short line segments) whose centers lie at the corners of an $L \times W$ rectangle. The 2 back wheels are parallel to the L-long rectangle sides. The 2 front wheels are oriented at angles θ_1, θ_2 respectively away from these two rectangle sides. What relationship should θ_1, θ_2 obey to make the car turn without sliding friction? Devise a planar mechanism with 1 degree of freedom to enforce this relationship. "More practical" mechanisms are better.

2. Given a 3-bar anchored linkage with anchored side length 2, other sides (in cyclic order) a,b,c, find conditions on a,b,c so that

1. Side a can rotate 360° about its anchorpoint, but side c can't
2. Both sides a,c can rotate 360°
3. Neither side a,c can

Does case 1 enable you to convert oscillating circular arc motion into continuous rotatory motion?

3. For a 3-bar symmetrical anchored linkage (fig. at right), Chebyshev wanted to know what relationship between a, b, h, w would cause the pen (third triangle vertex) to trace an "equiripple" approximate straight linecurve, i.e. a curve whose extrema on some x-interval were all equidistant in y from a horizontal line. Chebyshev's trigonometric and algebraic labors were immense. However, it turns out there is a much easier way to solve this problem by using *resultants* and *discriminants.* (I'll write a web subpage on resultants; also they are discussed in B.Van der Waerden: *Modern Algebra*, and the *Encyclopedia of Math.*) Can you accomplish this? (If you don't have access to an algebraic manipulation system, this will be too hard, but you can still outline the *strategy* of how to solve it.)

4. Consider the "life" cellular automaton [Berlekamp,Conway,Guy: Winning Ways (for your Mathematical Plays), Volume 2, 1982; BYTE: Sep 75, Oct 75, Dec 75, Jan 76, Dec 78, Jan 79, Apr 79, Oct 80, Jul 81; Scientific American: Apr 70, Oct 70, Nov 70, Jan 71, Feb 71, Mar 71, Apr 71, Nov 71, Jan 72, Dec 75, Mar 84, May 85, Feb 87, Aug 88, Aug 89, Sep 89, Jan 90; a ton of web sites]. This is a 2D square grid; each cell has 2 states "live" or "dead;" each cell has 8 immediate "neighbors;" the transition rules each time step are

1. If live cell has exactly 2 or 3 live neighbors it lives on next timestep, otherwise it dies.
2. If dead cell has exactly 3 live neighbors, it becomes live.

A proof is outlined in BCG 1982 that life is Turing universal because a computer may be built inside it that emulates a "counter machine." But counter machines suffer exponential slowdown relative to Turing machines. Can you figure out how to improve the universality proof to make it emulate a Turing or Von Neumann machine with only *polynomial* slowdown?

5. Consider 3×3 rotation matrices. Show that every rotation matrix R may be approximated arbitrarily closely by some product (e.g. $ABAAB$; the product depends on R and the accuracy demand) of two *fixed* rotation matrices A and B. Can this be generalized to $n \times n$ unitary matrices?

6. In my planar n-body problem in *classical* mechanics, suppose you want to initialize the bodies with momenta and positions accurate to P decimal places. Suppose you can manufacture planar mechanisms out of massless perfectly rigid rods and rivets (rods have exact integer lengths) at "cost" 1 per part you join on, and the n bodies can be positioned with the aid of your mechanism. (You and your friends move the mechanism by moving a finite number of specified gripping points with their hands. The hands are subject to thermal noise.) How much energy and "cost" will it take to do this initialization (upper and lower bounds)? How does this impact my "invalidation" of Church's thesis?!

7. True or false – for finite vs. infinite dimensional matrices?

1. $\exp(M) = I + M + M^2/2! + M^3/3! + \ldots$ converges.
2. $AB = I \;\Rightarrow\; BA = I$.
3. There exist A, B so that $\|I + AB - BA\| < 1$. (Here $\|M\| = \sup_{\vec{x},|\vec{x}|=1} |M\vec{x}|$.)