Homework problem set #1 for "Computing and the laws of physics."

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Due: day of 5th lecture of class (2 weeks after 1st; i.e. due Tues Oct 6)

[Rules: Consult any books, journals, etc you want. Acknowledge any sources, collaborations. If can't totally solve, try to take it as far as you can, and/or go for weaker results. Give arguments/proofs, not just answers like "3.6." Best answers will be presented in class by answerers, 1 class after the due date.]

[Purpose: to force you to learn/review some background knowledge, methods. Some questions have definite answers like 3c,5; while 1a,2b are more open ended.]

1. You've seen the "Rubik's $3 \times 3 \times 3$ cube." Consider an $N \times N \times N$ such cube R_N for any $N \geq 3$. Any "layer" can rotate any multiple of $90°$. When the cube is in its pristine state, you write distinct numbers from 1 to $6N^2$ on each of the $6N^2$ square faces of the surface cubies to make them distinct (as opposed to the wimpy approach of just considering any two same-color faces, as identical). As a function of N: how many configurations of R_N are there? (Note: getting the exact formula is pretty hard and not central to what I want you to learn from this problem. Getting fairly tight upper bounds without proving how tight, is comparatively easy.)
(a) A nasty person makes M moves (layer turns), scrambling your cube. Devise the best algorithm you can that, given M, N, and a description of the scrambled configuration, will find a path $\leq M$ moves long back to pristine. Are there faster ways to solve this problem than the "naive" method – if your computer has enough memory?
(b) What time and memory bounds of form $O(f(N, M))$ can you prove for your algorithm?
(c) Suppose your computer does 1 "step" per nanosecond. Estimate: how long to solve the $3 \times 3 \times 3$ cube using the naive algorithm and your best algorithm? If you had the age of the universe to compute – how large an N could you handle? How would this N be affected with 10 times *more* compute time?

2. "Church's thesis" (my interpretation, anyhow) says something like "Any reasonable computing device (and any reasonable physical system) can be simulated by an algorithm running on a Universal Turing Machine" (UTM). The "strong Church's thesis" continues "...with at most a constant increase in program size, and at most a polynomial slowdown factor."
(a) Demonstrate that a program running N steps on your desktop computer, i.e. a finite-wordsize RAM, [and consuming $\leq N$ contiguous words of memory] may be simulated by a UTM running for polynomial(N) steps. (You don't have to give every excruciating detail of the simulation algorithm and the UTM.)
(b) Did Alonzo Church ever actually say this? Try to track through the literature and find out where these Church theses came from and when!

3. (a) Prove: if H is a Hermitian matrix, then $\exp(iH)$ is a unitary matrix. (b) If U is unitary, may we then conversely conclude $i \ln U$ is Hermitian? (c) If A and B are Hermitian, which of the following are also Hermitian: AB, $A + B$, $A - B$, $AB - BA$, $AB + BA$, A^n (for each possible positive or negative integer n), ABA? (d) Suppose you've got a computer subroutine to find the eigenvalues/vectors of an arbitrary Hermitian matrix H – how could you use it to find the eigenvalues of a unitary matrix? (e) How could you use it to find $\arctan(H)$?

4. Suppose electrons are tiny hollow conductive spheres of radius r. Suppose the mass-energy $m_e c^2$ of the electron is the same as the classical E-field energy in the space surrounding the electron. What must r be? Would you expect the sphere to be rigid, or flexible? If the latter, estimate the energy of the first excited state caused by elastic waves. Has any such excited state been observed – and why or why not? Now [you probably want to steal results by V.Weisskopf: Phys Rev 56 (1939) 72-85, rather than attempting to do it yourself!] consider the electron's self-energy quantum mechanically, not classically. What, roughly, should r be to explain *this* energy? What about the excited states now? I once saw a popular talk by Ed Witten in which he argued that electrons could not be points or hollow spheres because the consequences were ludicrous. Therefore... electrons really are tiny loops (of radius r) of "string." What do you think of this argument? In what way might (or might not) strings help overcome what ludricrousness?

5. Consider a plane copper-insulator-copper sandwich. The left copper is charged a voltage V above the right (grounded). The reason the insulator is one is that an unbound electron in the insulator, if $V = 0$, would experience a potential energy penalty ϕ above what it would experience in the copper. The insulating layer is w wide. Solve Schrödinger's equation to determine the probability that an electron impinging as a normally oriented plane wave with kinetic energy E, would tunnel through, versus being reflected back from, the insulator. Explain how one could use formulas like this to deduce the current-voltage relation for 2-terminal electrical devices of this sort (don't actually do it, though – too hard). Warning: this explanation requires some knowledge of solid state physics, i.e. density of states in Bloch "conduction bands."