Instant Runoff Voting's Startling Rate of Failure

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--Abstract--

Instant Runoff Voting (IRV) is a sophisticated alternative voting system, designed to eliminate problems like vote-splitting and spoiler candidates which so often plague plurality-based elections. Instead of asking voters to name a single candidate, IRV asks for a ranked preference ballot, and uses these ballots to successively eliminate those candidates with the fewest first-place votes. IRV then redistributes votes according to preferences indicated on the ballots until some candidate receives a majority. While it is in many ways an improvement over traditional first-past-the-post elections, IRV nevertheless faces several unsettling problems, problems which are often concealed by the voting process. In some instances, IRV can display non-monotonicity, a particularly egregious drawback wherein ranking a candidate first can cause him to lose. This study uses a computer simulation to model likely election outcomes, and through this framework finds the proportion of non-monotonic IRV elections to be disturbingly large, large enough to severely question IRV's adequacy as a suitable election reform.

Burlington 2009

In March of 2009, the mayoral election in Burlington, VT was held using a method known as Instant Runoff Voting (IRV), a system often praised as a long-needed reform in public elections. This sophisticated voting system has many admirable qualities, most notably that it reduces the phenomenon of "vote-splitting", wherein candidates who share similar ideologies hurt one another in an election because their voter base is forced to choose between them. IRV deals with this problem by asking voters not only for their first choice, but for a ranked preference ballot. The system then successively eliminates those candidates who receive the fewest first place votes, and redistributes their votes according to the preferences expressed on the ballots, until one candidate has a majority of the votes. As a result, voters are less likely to "waste" their votes on a candidate who has little chance of winning, and are allowed to more fully express their true preferences. Proponents of Instant Runoff Voting (the most vocal of whom reside at the Center for Voting and Democracy) also praise the system for always selecting a majority candidate, and for effectively eliminating the "spoiler effect", wherein minor candidates can steal votes away from major candidates, altering the course of an election even if they themselves are incapable of winning.

On its surface, Instant Runoff voting seems to be a natural and reasonable improvement to our current system of plurality voting, and promises to resolve several of the issues that have dogged our public elections for centuries. And yet, looking deeper, there are several disturbing drawbacks to Instant Runoff Voting, drawbacks that seem to occur with startling frequency. To illustrate these drawbacks, let us take a look at the results from the Burlington election[4]:

1513	495	1289	1332	767	455	2043	371	568
W	W	W	M	M	M	K	K	K
M	K		K	W		M	W	
K	M		W	K		W	M	

The table above shows the vote tallies for the three major candidates during the final round of elimination (after two minor candidates had already been eliminated). Each column represents a possible ranked preference ballot, and the value at the top of each column denotes the number of ballots of that type which were submitted. ("W" denotes Kurt Wright, the Republican candidate. "M" denotes Andy Montroll, the Democrat. And "K" denotes Bob Kiss, the Progressive incumbent. The first column, for instance, denotes a ballot for any voter who prefers Wright over Montroll over Kiss.)

The Republican candidate, Kurt Wright, received 3297 first place votes. Andy Montroll, the Democratic candidate, received 2554 first place votes, and Bob Kiss, the Progressive incumbent, received 2982 first place votes. Wright receives a plurality of the votes, due to the fact that liberal voters are split between supporting the Democrat and the Progressive candidate. Under plurality, this vote-splitting would have resulted in a Republican victory. However, under IRV the candidate with the fewest first place votes is eliminated, and the second-place votes of his supporters are added to the other candidates' tallies. In this case, Montroll is eliminated, and the final vote totals are 4314 for Kiss and 4064 for Wright. Kiss wins, and the problem of vote-splitting is averted. Yet when we look closer at the data, several interesting conundrums begin to appear.

Firstly, notice how the voters who support Wright tend to pick Montroll as their second choice. It is clear that Republicans, should their first choice not be selected, would largely prefer the Democrat to the Progressive candidate. Yet, due to the successive elimination process of IRV, these preferences are never taken into consideration. Had Montroll gone head-to-head against Kiss, he might have won by a comfortable margin (4067 to 3477, assuming Republican voters would not have just stayed home). Yet in the three-way race, the voters that list Wright as their first choice are effectively marginalized by the system, and their full preferences are not taken into consideration. This is a problem that is not unique to the Burlington election. We would expect to find marginalized voters in every Instant Runoff Voting election, since there must always be a candidate who is not eliminated, but also does not win.

Montroll is what is known as the Condorcet winner for this election. That is to say, a majority of the voters prefer him to either of the other candidates based on the preference rankings given. Had Montroll entered a two-way race against either Wright or Kiss, he would have won each election (according to the original preference rankings) by a larger margin than that by which Kiss actually won. However, in the three-way race governed by IRV, Montroll loses, largely because the complete preferences of Republican voters are not taken into consideration.

Delving further, we find something even more curious, a paradox that is sufficient to completely undermine the legitimacy of the election's outcome. Notice what happens when the election profile presented above is altered so that Kiss is supported by even *more* voters:

1513	495	1289	1332	767	455	2043	371	568
W	W	W	M	M	M	K	K	K
M	K		K	W		M	W	
K	M		W	K		W	M	

↓

1513	195	836	1332	767	455	2043	671	1021
W	W	W	M	M	M	K	K	K
M	K		K	W		M	W	
K	M		W	K		W	M	

The first place vote totals are now Wright – 2544, Montroll – 2554, and Kiss – 3735. In this hypothetical election, Wright is eliminated instead of Montroll, and as a result Montroll wins the election 4067 to 3930.

This hypothetical may seem contrived, but its implications are disturbing. Remember, all we did to create this hypothetical ballot was to *increase* Kiss' support compared to the actual election, and as a result, he now *loses*. Even if this change is unlikely to actually occur, the mere existence of this companion election profile is enough to cast serious doubt on the legitimacy of Bob Kiss' mandate to govern. The fact that he only won the election because he received too few votes is a serious discredit to Instant Runoff Voting. This paradox is known as a violation of monotonicity.

Monotonicity, the condition that additional votes for a candidate should never hurt that candidate, and fewer votes should never help a candidate, is one of Kenneth Arrow's four classical preconditions for a fair and democratic voting system[1]. Instant Runoff Voting, as well as other systems which utilize similar types of successive elimination, are the only proposed voting systems known to violate this condition. Elections under IRV do not always violate monotonicity, and proponents of the system claim that it happens so rarely that it should not be considered a serious problem. Fairvote.org issues the following statement regarding violations of monotonicity[5]:

"In terms of the frequency of non-monotonicity in real-world elections: there is no evidence that this has ever played a role in any IRV election -- not the IRV presidential elections in Ireland, nor the literally thousands of hotly contested IRV federal elections that have taken place for generations in Australia, nor in any of the IRV elections in the United States."

It is a claim that deserves some consideration. If, in fact, non-monotonic outcomes are incredibly rare, then perhaps it is a problem of little import. This claim is, of course, very difficult to prove or refute without having access to the raw data from these actual IRV elections, which is generally very hard to come by. And, of course, this statement was written before the events in Burlington. What is needed is some way to gauge the prevalence of monotonicity violations in the absence of significant amounts of real world data. This is precisely what this paper attempts to accomplish.

Monotonicity in Instant Runoff Voting

Previous work by Robert Norman[6] has developed several formal concepts and definitions for determining if and when monotonicity is violated in an Instant Runoff election. Violations of monotonicity, he claims, can occur in two mirror-image situations:

- Violations of Monotonicity Type 1 (MT1) occur when, for a given election profile P with winning candidate A, there exists a companion profile P' with winning candidate B that can be generated by moving candidate A up in the rankings of at least one voter in P.
- Violations of Monotonicity Type 2 (MT2) occur when, for a given election profile P with winning candidate A, there exists a companion profile P' with winning candidate B that can be generated by moving candidate B down the rankings of at least one voter in P.

From these definitions, it follows that the companion profile P' for any election which violates MT1 must be a profile which violates MT2, and vice versa. In light of this fundamental correspondence, most of this paper will focus on violations of Monotonicity Type 1, with the understanding that each violation will always be paired with some set of profiles which violate Monotonicity Type 2.

Monotonicity Type 1

Let P be an election profile with three candidates A, B, and C, in which candidate C is eliminated under Instant Runoff Voting, and as a result, candidate A wins. Such a profile will violate monotonicity type 1 if and only if each candidate receives more than 25% of the first place votes. The proof for this is relatively straightforward. In order for a violation of MT1 to occur, candidate B must be eliminated by shifting first-place votes from B to A. If candidate C has 25% or fewer first-place votes, then candidate A will receive a majority before candidate B is eliminated, and thus monotonicity cannot be violated. Restricting candidate C to at least $\frac{n+3}{4}$ first place votes effectively prevents this.

In light of this, I shall define any election in which three candidates receive more than 25% of the first-place votes as a *close election*. Such elections not only satisfy the necessary condition for violations of Monotonicity Type 1, but they are also intrinsically interesting as an object of study, since the outcome of such elections is most uncertain, and thus determining how a voting system performs in such situations is of the utmost importance. Much of my subsequent analysis will focus on this subclass of elections.

It can further be shown that any close election *will* violate Monotonicity Type 1 should one of the following two conditions be met:

1) The Condorcet winner is not selected by Instant Runoff Voting. (This, as we noted before, is what occurred in the 2009 Burlington election).
2) The election profile exhibits a majority cyclic triple (i.e. the majority of voters prefer A to B, B to C, and C to A).

Manipulating election profile P so that candidate B is eliminated leaves candidates A and C running against one another in the final round, a contest which candidate C will win only win if one of the conditions above is met. If candidate C wins, then monotonicity type 1 is violated.

It has been asserted by voting theorist Nicolaus Tideman (personal communication from Robert Norman) that violations of monotonicity will be seen rarely in practice because such violations are fundamentally linked to the majority cyclic triple, an outcome which is seldom encountered in actual elections. This raises the question: what kinds of outcomes are likely to occur in Instant Runoff elections? And of these outcomes, what proportion will violate monotonicity?

Spatial Simulation – Rationale and Implementation

Although Instant Runoff Voting has been growing in popularity in recent years, raw data from actual IRV elections are still very difficult to come by, a frustrating fact for any voting theorist attempting to ascertain the empirical prevalence of monotonicity violations. In light of this dearth of data, there are several approaches that one could take.

As a first attempt, one could determine the proportion of *all* election profiles that exhibit violations of monotonicity. Robert Norman's studies, for instance, have discerned that anywhere from 15% to 25% of close election profiles (wherein no candidate receives more than 40% of the first-place vote) exhibit violations of monotonicity type 1. However, this approach fails to consider an important fact, one which Nicholas Tideman hints at in his commentary on majority cyclic triples – not all election profiles are equally likely. Given this, finding the proportion of all profiles that violate MT1 is not likely to yield a frequency that will correspond with any real-world findings. The challenge, therefore, is to determine which profiles *are* likely to occur in actual elections, and use this as the base population for studying MT1. Several methods have been proposed for achieving just such a feat[2]:

- Random Society Model – This model assumes that voters assign each candidate a "utility", and vote according to these values. When ranking candidates, voters will do so in order of decreasing utility. These values can be drawn from some random distribution (uniform, Gaussian, etc.).

- Spatial Models – Spatial models, like the one used in this study, can be derived from the following three assumptions:

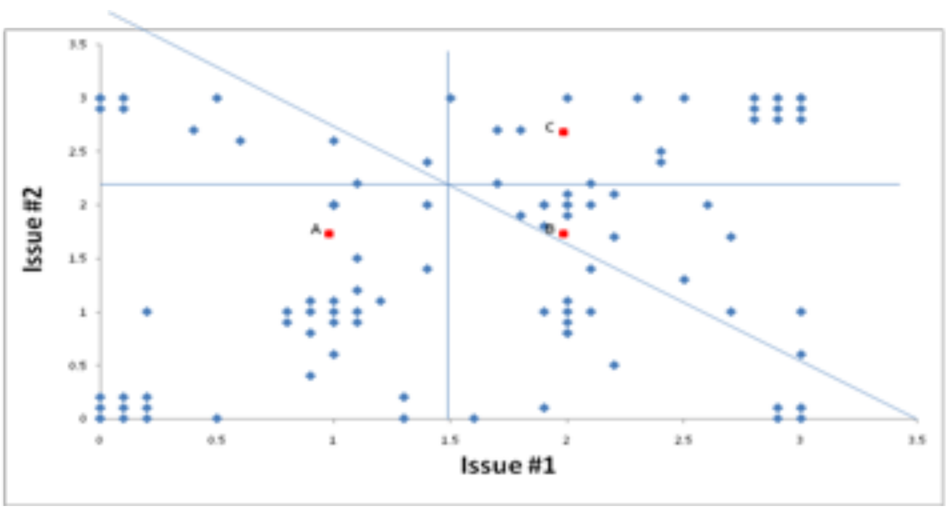
    - *Assumption #1* – Any candidate or voter can be represented as a vector in some *n*-dimensional "issue space". If we assume that opinions on any political issue can be expressed quantitatively on some arbitrary scale, then an *n*-dimensional vector is sufficient to quantitatively represent any voter or candidate's opinions over *n* separate issues.

    - *Assumption #2* – Variation in voter sentiment can be principally explained in a space of dimension less than *n*. Sentiment on political issues is often highly correlated (someone who opposes gay marriage, for instance, is highly likely to oppose legalized abortion as well), and as a consequence it can be sufficient to use a small number of proxy dimensions to illustrate variation in sentiment over a larger number of dimensions.

- *Assumption #3* – Voters most favor those candidates who are closest to them. When ranking candidates, voters will do so in reverse order of distance (both Euclidean and city block distance metrics are considered).

Using these assumptions it is possible to construct distributions of voter sentiment as a collection of multi-dimensional coordinates (for the purposes of this paper, we will confine our analysis to two and three dimensions). From this distribution we can hypothesize which election profiles will be most likely to occur, given similar random placements of candidates within the same issue space, and using this new population of likely election profiles, we can then ascertain the likely prevalence of monotonicity violations in real-world IRV elections.

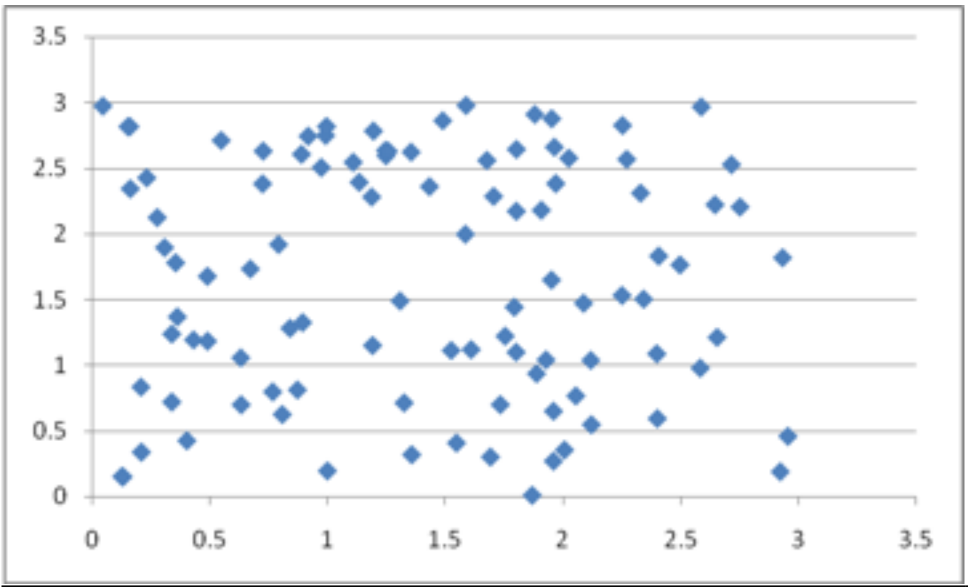
In the two-dimensional chart shown below, we can construct a hypothetical election profile by dividing the voter distribution into six regions. In the lower left-hand region, for instance, voters are closest to candidate A and farthest from candidate C. As such, we can assume that these voters will submit the ranked ballot A>B>C. We then repeat this process for all voters to construct the likely election profile.



Voter Distributions

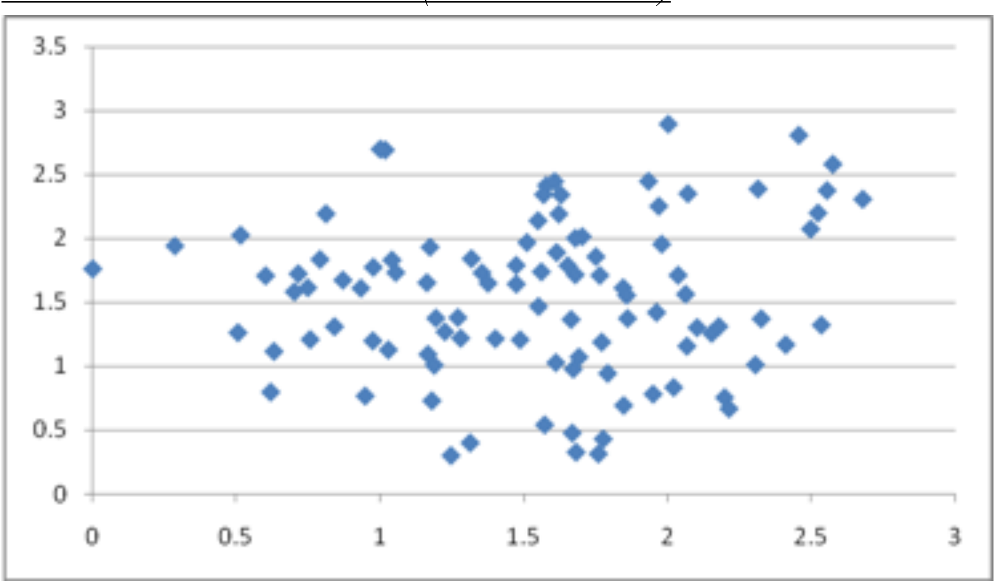
I constructed several two-dimensional voter distributions for use as inputs into the simulation. Each distribution contains 100 voters (results are not significantly altered when larger numbers of voters are used. 100 voters is sufficiently large to capture variation in voter sentiment, yet manageably small for purposes of visualization).

Uniform Distribution (Control Group)



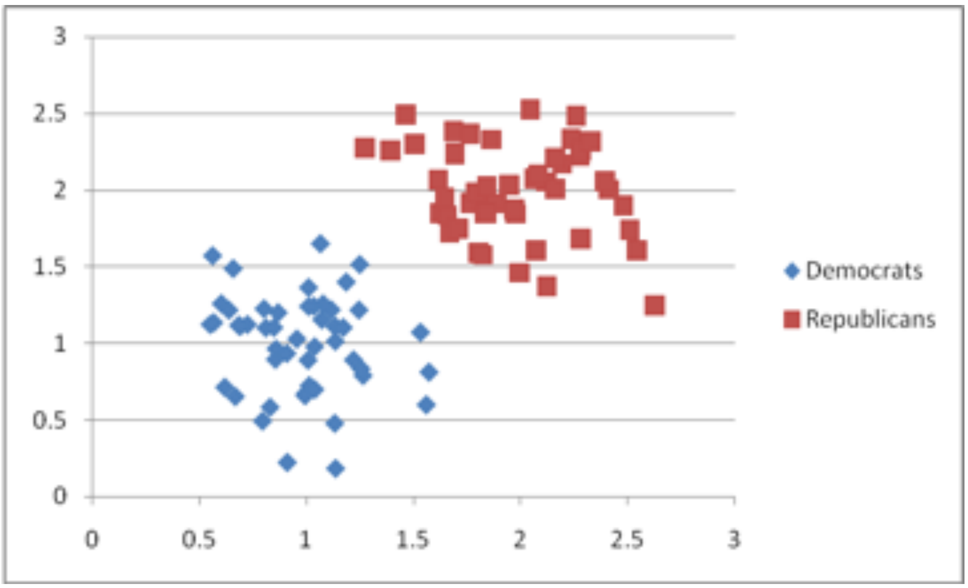
This distribution draws voter coordinates from a bivariate uniform distribution ranging from 0 to 3 (arbitrary values).

Bivariate Gaussian Distribution ("Centrist Voters")



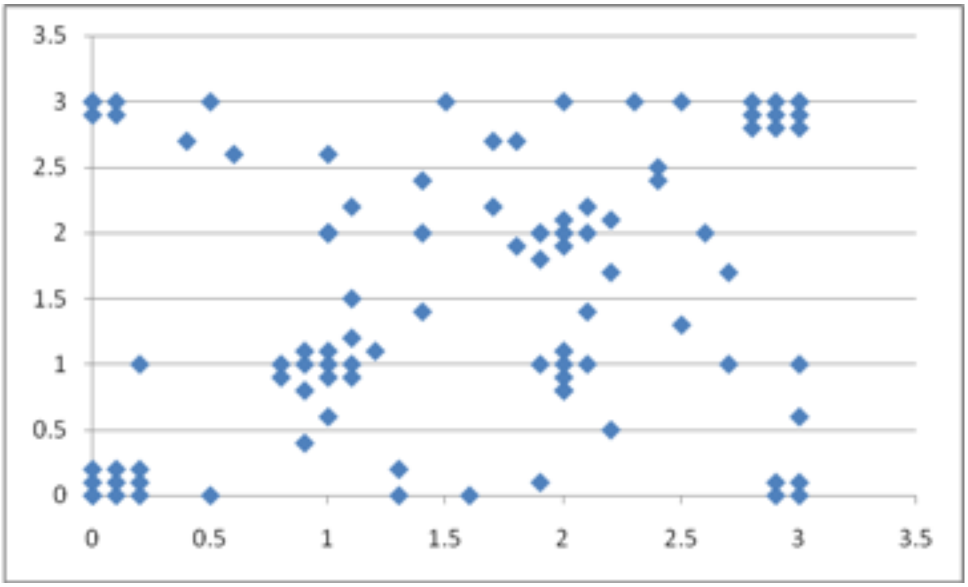
This distribution draws voter coordinates from a bivariate Gaussian distribution with mean 1.5 and standard deviation 0.6, effectively restricting values between 0 and 3 as above (again, by arbitrary convention).

Dual Bivariate Gaussian Distributions ("Polarized Voters")



This distribution draws voters from two separate bivariate Gaussian distributions, one representing a conservative and another representing a liberal population.

Multi-cluster Distribution ("Multi-polar Voters")



This distribution was chosen to reflect an electorate which is not clustered around moderate or polarized views (as in the bivariate Gaussian and polarized distributions above), but is instead divided into several smaller clusters.

Candidate Coordinates

For each simulated election, three candidate coordinates are selected using one of two methods:

1) Candidate coordinates are drawn from a bivariate Gaussian or uniform distribution, similar to the voter distributions. The standard deviation of this distribution is varied so that the candidates either perfectly mirror variation in public opinion, or stray very little from the center, as might be expected of candidates attempting to broaden their base.

2) Two candidate coordinates are drawn from the same distributions as the polarized voters. These two represent the major party candidates, and a third party candidate's coordinates are drawn from a bivariate uniform distribution (range 1 to 2).

Results

The results below are each based on 10,000-election trials.

Table 1: Uniform Voters (Control Group)

Candidate Generation Method	Type I Violations	Close Elections	Proportion
Uniform (Range 1 to 2)			
Trial 1	555	2407	23.06%
Trial 2	573	2408	23.80%
Trial 3	613	2513	24.39%
Uniform (Range 0 to 3)			
Trial 1	317	1497	21.18%
Trial 2	301	1491	20.19%
Trial 3	313	1549	20.21%
Gaussian (SD - 0.3)			
Trial 1	639	2564	24.92%
Trial 2	610	2541	24.01%
Trial 3	609	2620	23.24%
Gaussian (SD - 0.6)			
Trial 1	553	2331	23.72%
Trial 2	511	2188	23.35%
Trial 3	517	2133	24.24%
Democrat/Republican/Third Party			
Trial 1	1348	4407	30.59%
Trial 2	1301	4305	30.22%
Trial 3	1272	4358	29.19%

Table 2: Centrist Voters

Candidate Generation Method	Type I Violations	Close Elections	Proportion
Uniform (Range 1 to 2)			
Trial 1	817	3225	25.33%
Trial 2	769	3132	24.55%
Trial 3	819	3171	25.83%
Uniform (Range 0 to 3)			
Trial 1	97	556	17.45%
Trial 2	80	578	13.84%
Trial 3	77	544	14.15%
Gaussian (SD - 0.3)			
Trial 1	773	3015	25.64%
Trial 2	785	3082	25.47%

Trial 3	743	3123	23.79%
Gaussian (SD - 0.6)			
Trial 1	215	1131	19.01%
Trial 2	199	1087	18.31%
Trial 3	171	1078	15.86%
Democrat/Republican/Third Party			
Trial 1	483	2551	18.93%
Trial 2	434	2486	17.46%
Trial 3	430	2596	16.56%

Table 3: Polarized Voters

Candidate Generation Method	Type I Violations	Close Elections	Proportion
Uniform (Range 1 to 2)			
Trial 1	154	735	20.95%
Trial 2	174	796	21.86%
Trial 3	175	778	22.49%
Uniform (Range 0 to 3)			
Trial 1	52	331	15.71%
Trial 2	60	344	17.44%
Trial 3	61	329	18.54%
Gaussian (SD - 0.3)			
Trial 1	145	763	19.00%
Trial 2	183	810	22.59%
Trial 3	143	703	20.34%
Gaussian (SD - 0.6)			
Trial 1	119	514	23.15%
Trial 2	104	540	19.26%
Trial 3	105	554	18.95%
Democrat/Republican/Third Party			
Trial 1	187	933	20.04%
Trial 2	217	963	22.53%
Trial 3	178	937	19.00%

Table 4: Multi-polar Voters

Candidate Generation Method	Type I Violations	Close Elections	Proportion
Uniform (Range 1 to 2)			
Trial 1	400	1563	25.59%
Trial 2	381	1555	24.50%
Trial 3	397	1555	25.53%
Uniform (Range 0 to 3)			
Trial 1	208	1057	19.68%
Trial 2	190	1041	18.25%
Trial 3	221	1069	20.67%
Gaussian (SD - 0.3)			
Trial 1	450	1702	26.44%
Trial 2	442	1637	27.00%
Trial 3	438	1638	26.74%
Gaussian (SD - 0.6)			
Trial 1	390	1520	25.66%
Trial 2	406	1641	24.74%
Trial 3	419	1699	24.66%
Democrat/Republican/Third Party			
Trial 1	1133	2621	43.23%
Trial 2	1125	2644	42.55%
Trial 3	1151	2587	44.49%

Startling Proportions

As you can see from the simulation results in tables 1 through 4, the proportion of close election profiles which violate Monotonicity Type 1 is truly startling, ranging from 13.84% when the electorate exhibits centrist tendencies and the candidates are chosen completely at random, to 44.49% when the electorate is clustered into multiple ideological groups, and there are two major party candidates located near the main clusters and a third party candidate placed randomly. 13.84% of close elections is itself an unreasonable proportion, yet it is particularly noteworthy that the proportion of close elections which violate Monotonicity Type 1 gets larger when more realistic assumptions are built into the model (polarized electorate, central candidate placement, candidates locating near clusters).

Notice how, when the electorate is split perfectly into two ideological clusters (polarized voters), close elections with a two major party candidates and a third party candidate have a 1-

in-5 chance of violating monotonicity. This result alone is enough to cast serious doubt on the efficacy of IRV for deciding close elections. Even more disconcerting, however, is what happens when we do not simply have one or two clusters of voters, but several (the "multi-polar distribution"). Doing so causes the proportion of violations in close elections to more than double!

Cyclicity

At first glance, it seems that the proportions of Type 1 violations derived from the simulation are remarkably similar to the proportions from all close election profiles. Could it be that the simulation is simply replicating the kind of analysis we see in Robert Norman's studies, and is not actually outputting a subset of likely election profiles at all? To answer this question, let us examine the prevalence of simulated election profiles which exhibit majority cyclic triples. As noted previously, when looking at all election profiles, majority cyclic triples occur in more than half of the profiles which violate monotonicity type 1, leading many to claim that such profiles represent an unrealistically large component of monotonicity violations. However, when looking only at simulated profiles, the proportion of those election profiles which are simultaneously cyclic and violate monotonicity type 1 is much smaller than the proportion of all profiles (on average roughly 3%; see tables 5 through 8). Cyclic majority triples never comprise more than 1.5% of all likely profiles produced by the simulation.

Not only does the proportion of violations among likely profiles remain as high if not higher than the proportion among all profiles, but it does so without relying upon majority cyclic triples, which the simulation confirms is a highly unlikely outcome. As such, the prevalence of monotonicity violations of type 1 is intricately linked not to the majority cyclic triple, but to IRV's Condorcet selection efficiency in close elections.

Table 5: Cyclic Violations in Uniform Voter Elections

	Cyclic Violations	Type I Violations	Cyclic Proportion
Uniform (Range 1 to 2)			
Trial 1	19	555	3.42%
Trial 2	14	573	2.44%
Trial 3	22	613	3.59%
Uniform (Range 0 to 3)			
Trial 1	2	317	0.63%
Trial 2	3	301	1.00%
Trial 3	5	313	1.60%
Gaussian (SD - 0.3)			
Trial 1	12	639	1.88%
Trial 2	23	610	3.77%
Trial 3	10	609	1.64%
Gaussian (SD - 0.6)			
Trial 1	6	553	1.08%
Trial 2	5	511	0.98%
Trial 3	3	517	0.58%
Democrat/Republican/Third Party			
Trial 1	6	1348	0.45%
Trial 2	0	1301	0.00%
Trial 3	5	1272	0.39%

Table 6: Cyclic Violations in Centrist Voter Elections

	Cyclic Violations	Type I Violations	Cyclic Proportion
Uniform (Range 1 to 2)			
Trial 1	58	817	7.10%
Trial 2	58	769	7.54%
Trial 3	56	819	6.84%
Uniform (Range 0 to 3)			
Trial 1	10	97	10.31%
Trial 2	11	80	13.75%
Trial 3	5	77	6.49%
Gaussian (SD - 0.3)			
Trial 1	58	773	7.50%
Trial 2	51	785	6.50%
Trial 3	34	743	4.58%

Gaussian (SD - 0.6)			
Trial 1	17	215	7.91%
Trial 2	18	199	9.05%
Trial 3	10	171	5.85%
Democrat/Republican/Third Party			
Trial 1	24	483	4.97%
Trial 2	28	434	6.45%
Trial 3	23	430	5.35%

Table 7: Cyclic Violations in Polarized Voter Elections

Candidate Generation Method	Cyclic Violations	Type I Violations	Cyclic Proportion
Uniform (Range 1 to 2)			
Trial 1	1	154	0.65%
Trial 2	1	174	0.57%
Trial 3	1	175	0.57%
Uniform (Range 0 to 3)			
Trial 1	0	52	0.00%
Trial 2	0	60	0.00%
Trial 3	0	61	0.00%
Gaussian (SD - 0.3)			
Trial 1	4	145	2.76%
Trial 2	5	183	2.73%
Trial 3	4	143	2.80%
Gaussian (SD - 0.6)			
Trial 1	1	119	0.84%
Trial 2	0	104	0.00%
Trial 3	0	105	0.00%
Democrat/Republican/Third Party			
Trial 1	0	187	0.00%
Trial 2	0	217	0.00%
Trial 3	0	178	0.00%

Table 8: Cyclic Violations in Multi-polar Voter Elections

Candidate Generation Method	Cyclic Violations	Type I Violations	Cyclic Proportion

Uniform (Range 1 to 2)			
Trial 1	21	400	5.25%
Trial 2	8	381	2.10%
Trial 3	11	397	2.77%
Uniform (Range 0 to 3)			
Trial 1	8	208	3.85%
Trial 2	2	190	1.05%
Trial 3	4	221	1.81%
Gaussian (SD - 0.3)			
Trial 1	14	450	3.11%
Trial 2	18	442	4.07%
Trial 3	19	438	4.34%
Gaussian (SD - 0.6)			
Trial 1	5	390	1.28%
Trial 2	8	406	1.97%
Trial 3	7	419	1.67%
Democrat/Republican/Third Party			
Trial 1	13	1133	1.15%
Trial 2	5	1125	0.44%
Trial 3	15	1151	1.30%

Monotonicity Type 2

I have tended to de-emphasize monotonicity type 2 up to now for two reasons. For one, whereas the necessary condition for a violation of monotonicity type 1 is inherently appealing (close elections are, in and of themselves, an interesting object of study), the necessary condition for a violation of Type 2 is somewhat more artificial ($b + c \geq 2a + 2$). Secondly, since the proportions of Type 1 violations are so startlingly large, and the proportions of Type 2 violations relatively small in comparison, it adds little emphasis to discuss such violations. However, in the interest of completeness, the following charts describe the overall proportions of Type 2 violations among the likely election profiles produced by the simulation (again, based on 10,000 election trials).

Table 9: Type II Violations in Uniform Voter Elections

Candidate Generation Method	Type II Violations	Potential Type II's	Proportion
Uniform (Range 1 to 2)			
Trial 1	79	574	13.76%
Trial 2	75	570	13.16%
Trial 3	64	548	11.68%
Uniform (Range 0 to 3)			
Trial 1	38	340	11.18%
Trial 2	36	409	8.80%
Trial 3	41	378	10.85%
Gaussian (SD - 0.3)			
Trial 1	87	504	17.26%
Trial 2	86	520	16.54%
Trial 3	102	513	19.88%
Gaussian (SD - 0.6)			
Trial 1	60	640	9.38%
Trial 2	54	676	7.99%
Trial 3	51	623	8.19%
Democrat/Republican/Third Party			
Trial 1	121	1343	9.01%
Trial 2	110	1291	8.52%
Trial 3	116	1284	9.03%

Table 10: Type II Violations in Centrist Voter Elections

Candidate Generation Method	Type II Violations	Potential Type II's	Proportion
Uniform (Range 1 to 2)			
Trial 1	60	767	7.82%
Trial 2	74	803	9.22%
Trial 3	74	808	9.16%
Uniform (Range 0 to 3)			
Trial 1	16	97	16.49%
Trial 2	20	99	20.20%
Trial 3	15	103	14.56%
Gaussian (SD - 0.3)			
Trial 1	74	663	11.16%
Trial 2	78	678	11.50%

Trial 3	82	681	12.04%
Gaussian (SD - 0.6)			
Trial 1	26	234	11.11%
Trial 2	27	236	11.44%
Trial 3	22	210	10.48%
Democrat/Republican/Third Party			
Trial 1	60	534	11.24%
Trial 2	42	466	9.01%
Trial 3	51	513	9.94%

Table 11: Type II Violations in Polarized Voter Elections

Candidate Generation Method	Type II Violations	Potential Type II's	Proportion
Uniform (Range 1 to 2)			
Trial 1	12	386	3.11%
Trial 2	12	435	2.76%
Trial 3	14	385	3.64%
Uniform (Range 0 to 3)			
Trial 1	5	159	3.14%
Trial 2	2	148	1.35%
Trial 3	7	165	4.24%
Gaussian (SD - 0.3)			
Trial 1	11	385	2.86%
Trial 2	15	386	3.89%
Trial 3	8	386	2.07%
Gaussian (SD - 0.6)			
Trial 1	3	269	1.12%
Trial 2	5	289	1.73%
Trial 3	6	328	1.83%
Democrat/Republican/Third Party			
Trial 1	0	848	0.00%
Trial 2	0	865	0.00%
Trial 3	1	850	0.12%

Table 12: Type II Violations in Multi-Polar Voter Elections

Candidate Generation Method	Type II Violations	Potential Type II's	Proportion
Uniform (Range 1 to 2)			
Trial 1	85	923	9.21%
Trial 2	83	962	8.63%
Trial 3	86	967	8.89%
Uniform (Range 0 to 3)			
Trial 1	43	448	9.60%
Trial 2	56	497	11.27%
Trial 3	51	466	10.94%
Gaussian (SD - 0.3)			
Trial 1	88	843	10.44%
Trial 2	81	786	10.31%
Trial 3	93	785	11.85%
Gaussian (SD - 0.6)			
Trial 1	74	681	10.87%
Trial 2	70	712	9.83%
Trial 3	65	700	9.29%
Democrat/Republican/Third Party			
Trial 1	51	868	5.88%
Trial 2	78	944	8.26%
Trial 3	64	901	7.10%

Looking Forward

Bear in mind that this paper by no means advocates a return to plurality voting. Plurality, with its flaws like spoilers and split votes is certainly no better than IRV. Plurality with runoff is similarly unimpressive, as it has all the flaws of IRV, but such problems are more difficult to detect since full preference rankings are not recorded. There are other systems of voting which reasonably eliminate the problems of vote-splitting, yet are entirely monotonic. The Borda

Count, for instance, also uses ranked ballots, assigning points to each candidate based on how high they are in each voter's rankings. Approval voting deals with the problem a different way, by allowing voters to cast votes for as many candidates as they wish. Each of these systems, of course, has problems of its own, but nothing nearly as insidious and backwards as those found in Instant Runoff Voting. If we are to effectively adopt voting system reform, then the public must be made fully aware of all the benefits and drawbacks of each proposed system, so it can most effectively decide which values to incorporate into our system of democracy.

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Appendix 1 – Results using City Block Distance Metric

Table 1: Uniform Voters

Candidate Generation Method	Type I Violations	Close Elections	Proportion
Uniform (Range 1 to 2)	516	2151	23.99%

-	505	2199	22.96%
-	486	2184	22.25%
Uniform (Range 0 to 3)	373	1647	22.65%
-	376	1731	21.72%
-	372	1685	22.08%
Gaussian (SD - 0.3)	486	2434	19.97%
	462	2400	19.25%
	455	2357	19.30%
Gaussian (SD - 0.6)	553	2386	23.18%
	543	2233	24.32%
	557	2319	24.02%
Democrat/Republican/Third Party	1127	4390	25.67%
	1112	4423	25.14%
	1151	4525	25.44%

Table 2: Centrist Voters

Candidate Generation Method	Type I Violations	Close Elections	Proportion
Uniform (Range 1 to 2)	846	3109	27.21%
-	832	3166	26.28%
-	839	3130	26.81%
Uniform (Range 0 to 3)	83	559	14.85%
-	91	524	17.37%
-	90	532	16.92%
Gaussian (SD - 0.3)	579	2907	19.92%
	617	2868	21.51%
	588	2916	20.16%
Gaussian (SD - 0.6)	191	1159	16.48%
	180	1091	16.50%
	181	1152	15.71%
Democrat/Republican/Third Party	417	2599	16.04%
	421	2629	16.01%
	414	2576	16.07%

Table 3: Polarized Voters

Candidate Generation Method	Type I Violations	Close Elections	Proportion
Uniform (Range 1 to 2)	197	565	34.87%
-	208	536	38.81%
-	189	558	33.87%
Uniform (Range 0 to 3)	98	359	27.30%
-	85	377	22.55%

-	53	314	16.88%
Gaussian (SD - 0.3)	136	378	35.98%
	141	366	38.52%
	157	389	40.36%
Gaussian (SD - 0.6)	166	565	29.38%
	193	590	32.71%
	157	526	29.85%
Democrat/Republican/Third Party	235	1085	21.66%
	250	1030	24.27%
	272	1122	24.24%

Table 4: Multi-polar Voters

Candidate Generation Method	Type I Violations	Close Elections	Proportion
Uniform (Range 1 to 2)	195	745	26.17%
-	184	728	25.27%
-	184	755	24.37%
Uniform (Range 0 to 3)	296	1518	19.50%
-	286	1532	18.67%
-	261	1534	17.01%
Gaussian (SD - 0.3)	142	608	23.36%
	164	632	25.95%
	146	578	25.26%
Gaussian (SD - 0.6)	390	1789	21.80%
	430	1712	25.12%
	382	1737	21.99%
Democrat/Republican/Third Party	817	1880	43.46%
	797	1859	42.87%
	815	1886	43.21%

Table 5: Cyclic Violations in Uniform Voter Elections

Candidate Generation Method	Cyclic Violations	Type I Violations	Proportion
Uniform (Range 1 to 2)	49	516	9.50%
-	41	505	8.12%
-	38	486	7.82%
Uniform (Range 0 to 3)	38	373	10.19%
-	48	376	12.77%
-	43	372	11.56%
Gaussian (SD - 0.3)	29	486	5.97%

	24	462	5.19%
	21	455	4.62%
Gaussian (SD - 0.6)	42	553	7.59%
	51	543	9.39%
	47	557	8.44%
Democrat/Republican/Third Party	66	1127	5.86%
	49	1112	4.41%
	48	1151	4.17%

Table 6: Cyclic Violations in Centrist Voter Elections

Candidate Generation Method	Cyclic Violations	Type I Violations	Proportion
Uniform (Range 1 to 2)	120	846	14.18%
-	104	832	12.50%
-	103	839	12.28%
Uniform (Range 0 to 3)	26	83	31.33%
-	28	91	30.77%
-	26	90	28.89%
Gaussian (SD - 0.3)	71	579	12.26%
	61	617	9.89%
	60	588	10.20%
Gaussian (SD - 0.6)	32	191	16.75%
	27	180	15.00%
	30	181	16.57%
Democrat/Republican/Third Party	69	417	16.55%
	66	421	15.68%
	64	414	15.46%

Table 7: Cyclic Violations in Polarized Voter Elections

Candidate Generation Method	Cyclic Violations	Type I Violations	Proportion
Uniform (Range 1 to 2)	12	197	6.09%
-	11	208	5.29%
-	13	189	6.88%
Uniform (Range 0 to 3)	17	98	17.35%
-	12	85	14.12%
-	6	53	11.32%
Gaussian (SD - 0.3)	4	136	2.94%
	4	141	2.84%
	5	157	3.18%

Gaussian (SD - 0.6)	7	166	4.22%
	8	193	4.15%
	8	157	5.10%
Democrat/Republican/Third Party	1	235	0.43%
	0	250	0.00%
	0	272	0.00%

Table 8: Cyclic Violations in Multi-polar Voter Elections

Candidate Generation Method	Cyclic Violations	Type I Violations	Proportion
Uniform (Range 1 to 2)	18	195	9.23%
-	10	184	5.43%
-	15	184	8.15%
Uniform (Range 0 to 3)	13	296	4.39%
-	12	286	4.20%
-	12	261	4.60%
Gaussian (SD - 0.3)	1	142	0.70%
	1	164	0.61%
	0	146	0.00%
Gaussian (SD - 0.6)	7	390	1.79%
	7	430	1.63%
	5	382	1.31%
Democrat/Republican/Third Party	6	817	0.73%
	5	797	0.63%
	2	815	0.25%

Appendix 2 – Results using a Three-Dimensional Issue Space

Table 1: Uniform Voters

Candidate Generation Method	Type I Violations	Close Elections	Proportion
Uniform (Range 1 to 2)			
Trial 1	252	1520	16.58%
Trial 2	207	1588	13.04%
Trial 3	239	1585	15.08%
Uniform (Range 0 to 3)			
Trial 1	823	3836	21.45%
Trial 2	839	3845	21.82%
Trial 3	813	3863	21.05%
Gaussian (SD - 0.3)			
Trial 1	896	3994	22.43%

Trial 2	847	3881	21.82%
Trial 3	884	3988	22.17%
Gaussian (SD - 0.6)			
Trial 1	362	2309	15.68%
Trial 2	380	2402	15.82%
Trial 3	363	2373	15.30%
Democrat/Republican/Third Party			
Trial 1	989	4746	20.84%
Trial 2	964	4692	20.55%
Trial 3	946	4774	19.82%

Table 2: Centrist Voters

Candidate Generation Method	Type I Violations	Close Elections	Proportion
Uniform (Range 1 to 2)			
Trial 1	649	3527	18.40%
Trial 2	719	3606	19.94%
Trial 3	658	3586	18.35%
Uniform (Range 0 to 3)			
Trial 1	83	687	12.08%
	75	673	11.14%
Trial 3	79	679	11.63%
Gaussian (SD - 0.3)			
Trial 1	601	3299	18.22%
Trial 2	576	3300	17.45%
Trial 3	610	3320	18.37%
Gaussian (SD - 0.6)			
Trial 1	107	960	11.15%
Trial 2	111	931	11.92%
Trial 3	123	957	12.85%
Democrat/Republican/Third Party			
Trial 1	153	1659	9.22%
Trial 2	174	1739	10.01%
Trial 3	155	1766	8.78%

Table 3: Polarized Voters

Candidate Generation Method	Type I Violations	Close Elections	Proportion
Uniform (Range 1 to 2)			
Trial 1	190	829	22.92%

Trial 2	204	828	24.64%
Trial 3	195	851	22.91%
Uniform (Range 0 to 3)			
Trial 1	85	393	21.63%
Trial 2	91	356	25.56%
Trial 3	96	403	23.82%
Gaussian (SD - 0.3)			
Trial 1	200	824	24.27%
Trial 2	204	752	27.13%
Trial 3	188	814	23.10%
Gaussian (SD - 0.6)			
Trial 1	110	476	23.11%
Trial 2	126	527	23.91%
Trial 3	124	492	25.20%
Democrat/Republican/Third Party			
Trial 1	142	467	30.41%
Trial 2	161	498	32.33%
Trial 3	142	444	31.98%

Table 4: Cyclic Violations in Uniform Voter Elections

Candidate Generation Method	Cyclic Violations	Type I Violations	Proportion
Uniform (Range 1 to 2)			
Trial 1	21	252	8.33%
Trial 2	18	207	8.70%
Trial 3	13	239	5.44%
Uniform (Range 0 to 3)			
Trial 1	51	823	6.20%
Trial 2	63	839	7.51%
Trial 3	63	813	7.75%
Gaussian (SD - 0.3)			
Trial 1	43	896	4.80%
Trial 2	41	847	4.84%
Trial 3	45	884	5.09%
Gaussian (SD - 0.6)			
Trial 1	11	362	3.04%
Trial 2	18	380	4.74%
Trial 3	12	363	3.31%

Democrat/Republican/Third Party			
Trial 1	17	989	1.72%
Trial 2	16	964	1.66%
Trial 3	14	946	1.48%

Table 5: Cyclic Violations in Centrist Voter Elections

Candidate Generation Method	Cyclic Violations	Type I Violations	Proportion
Uniform (Range 1 to 2)			
Trial 1	53	649	8.17%
Trial 2	46	719	6.40%
Trial 3	55	658	8.36%
Uniform (Range 0 to 3)			
Trial 1	3	83	3.61%
	7	75	9.33%
Trial 3	5	79	6.33%
Gaussian (SD - 0.3)			
Trial 1	27	601	4.49%
Trial 2	24	576	4.17%
Trial 3	37	610	6.07%
Gaussian (SD - 0.6)			
Trial 1	5	107	4.67%
Trial 2	7	111	6.31%
Trial 3	5	123	4.07%
Democrat/Republican/Third Party			
Trial 1	5	153	3.27%
Trial 2	5	174	2.87%
Trial 3	5	155	3.23%

Table 6: Cyclic Violations in Polarized Voter Elections

Candidate Generation Method	Cyclic Violations	Type I Violations	Proportion
Uniform (Range 1 to 2)			
Trial 1	8	190	4.21%
Trial 2	10	204	4.90%
Trial 3	8	195	4.10%
Uniform (Range 0 to 3)			
Trial 1	0	85	0.00%
Trial 2	1	91	1.10%
Trial 3	0	96	0.00%
Gaussian (SD - 0.3)			
Trial 1	4	200	2.00%

Trial 2	6	204	2.94%
Trial 3	4	188	2.13%
Gaussian (SD - 0.6)			
Trial 1	1	110	0.91%
Trial 2	0	126	0.00%
Trial 3	0	124	0.00%
Democrat/Republican/Third Party			
Trial 1	0	142	0.00%
Trial 2	0	161	0.00%
Trial 3	0	142	0.00%