Is Majority Rule the Best Election Method?

Eric Maskin

SEPTEMBER 2001, PAPER NUMBER 11

The Occasional Papers of the School of Social Science are normally versions of talks given at the School's weekly Thursday Seminar. At these seminars, Members present work-in-progress and then take questions. There is often lively conversation and debate, some of which will be included with the papers. We have chosen papers we thought would be of interest to a broad audience. Our aim is to capture some part of the cross-disciplinary conversations that are the mark of the School's programs. While members are drawn from specific disciplines of the social sciences—anthropology, economics, sociology and political science—as well as history, philosophy, literature and law, the School encourages new approaches that arise from exposure to different forms of interpretation. The papers in this series differ widely in their topics, methods, and disciplines. Yet they concur in a broadly humanistic attempt to understand how—and under what conditions—the concepts that order experience in different cultures and societies are produced, and how they change.

Eric Maskin is the A.O. Hirschman Professor of Social Science at the Institute for Advanced Study. He is an economist whose papers touch on many issues of broad interest to the social sciences and have often had important policy implications. He has worked in many areas of economic theory, including game theory, the economics of incentives, and social choice theory. His current projects involve designing auctions, comparing different electoral rules (such as majority voting), understanding the role of monetary policy, and studying both the advantages and drawbacks of protecting intellectual property. He is editor of the journal *Economic Letters*.

The paper printed here, *Is Majority Rule the Best Election Method?*, is the text of the talk that Maskin gave in the Faculty Lecture Series. The Faculty Lecture Series is an important part of the Institute's program of community outreach: one member of each of the four Institute faculties gives a public lecture to the larger community each year. Like the Thursday seminars, these lectures make time for questions and discussion. A reception follows in which Institute and community members meet for more individualized conversations. In presenting this Occasional Paper, we have maintained the more informal style of Professor Maskin's talk.

Because the lecture was originally delivered on October 25, 2000, two weeks before the disputed presidential election, Maskin could not have known how timely his reflections were. While the version we present here is somewhat edited, the relevance of the questions required no updating.

Is Majority Rule the Best Election Method?

1. Two Questions in Democratic Theory

It has probably not escaped your attention that there will be a presidential election in a couple of weeks [Note: the election was held on November 7, 2000]. But you can relax: I have no intention of providing any sort of analysis of that particular election. Still, I thought this might be an opportune time to stand back and reflect on the institution of elections in general. Specifically, I'd like to consider two fundamental questions for democratic theorists. First, why should we have elections at all? And second, assuming that we do have them, how should they be conducted? (In other words, what should the rules of the election be?) After all, we have lots of options: majority rule, or plurality rule, or something else. Needless to say, the answers to both questions are important.

I think it is not too difficult to provide an uncontroversial answer to the first: elections, at least in democratic societies, are a way of resolving major public decisions, such as who will be president, in a way that reflects the views of members of society. In fact, it is difficult to think of a way of ensuring that the citizenry is represented in public decision making without holding elections (here I include referenda under the heading of elections).

Now, the answer to the second question—which might be rephrased as "How should we conduct elections to ensure that public decisions best represent individual citizens' views?"—is harder. But that's the question that I want to focus on today.

What sort of election mechanism we ought to use would be unproblematic if everybody in society had the same views. Just about any election method would work in that case because any election method that is even vaguely reasonable will satisfy what we might call the consensus principle. This principle requires that if everybody in society agrees candidate A is better than candidate B, then B will not get elected. An election procedure would, I venture to say, have to be pretty pathological to result in the election of B. So I think we can reasonably restrict attention to procedures that satisfy this consensus principle. And clearly any of them would do if everyone had the same views on the candidates. In reality, of course, people differ in their views, and so let's turn to that case.

Let's begin with a simple example. Let's imagine that there is a voter named Al (see Table 1) who happens to prefer Gore to Nader, Nader to Bush, and Bush to Buchanan [Note: Al Gore, Ralph Nader, George W. Bush, and Pat Buchanan were the four principal candidates in the 2000 Presidential election]. There's another voter named W who prefers Bush to Nader, Nader to Buchanan, and Buchanan to Gore, i.e., he has the ranking Bush, Nader, Buchanan, Gore (see Table 2). Let's suppose, since we're in the realm of hypothesis anyway, that everybody in society is either like Al or like W. In fact, let's imagine that 60% have Al's views, and 40% have W's views (see Table 3). The question is: What is the right outcome for this distribution of preferences? Who should be elected in this case? Under an electoral system governed by plurality or majority rule (in which a candidate wins if he or she garners, respectively, a plurality or majority of all votes cast), it's clear what would happen: Gore would win in a landslide because 60% of the electorate prefer him to anybody else. We might ask, however, whether this outcome truly

Al's Ranking
Gore
Nader
Bush
Buchanan

Table 1

W's Ranking
Bush
Nader
Buchanan
Gore

Table 2

reflects the views of the electorate. To answer the question, we must say what "truly reflects" means. To explore that issue, let's compare the plurality- or majority-rule outcome with what we would get under another commonly used voting mechanism called rank order voting.

2. Majority Rule versus Rank Order Voting

Rank order voting is a particularly popular procedure among committees. Here's how it works: If there are four candidates for election, each voter assigns four points to his or her favorite candidate, three to the next favorite, two to the next, one to the least favorite. The points are added up for each candidate, and the winner is the candidate with the biggest total. I should mention that, as far as we can tell, rank order voting was invented by the eighteenth-century French engineer, Jean-Charles Borda. And so among voting theorists, rank order voting is sometimes called the Borda count. But notice that for the voting population of Table 3, it gives rise to quite a different outcome from majority voting. If there are a hundred voters in all, then, according to this table, sixty voters place Gore first. This means that Gore will get 240 points—that is 60 times 4 points—from these voters. And since 40 voters place Gore last, he'll get an additional 40 points from those voters for a grand total of 280 points.

60%	40%
Gore	Bush
Nader	Nader
Bush	Buchanan
Buchanan	Gore

Table 3

We can go through the point computations for the other candidates as well: Bush gets 280; Buchanan gets 140. Strikingly, Nader ends up with 300 points, even though nobody places him first. But he is a consistent second, and that is good enough to elect him under rank order voting.

So majority/plurality rule and rank order voting result in sharply different outcomes. Literally nobody would vote for Nader in a system in which voters simply express their favorite choices (the system that is used in practice for presidential and congressional elections). But Nader wins fairly handily under rank order voting. Given this contrast, what can we say about which voting mechanism does a better job of representing voters' views? I think that one way to answer this question is to go back to fundamental principles. Let us try to formulate the principles or "axioms" that any good democratic electoral mechanism ought to satisfy. I've already mentioned one of those axioms, the consensus principle. But this principle doesn't distinguish between majority rule and rank order voting; both voting methods clearly satisfy it.

Another important democratic principle is the idea that all voters should count equally in the voting process. This is sometimes called the "one-man, one-vote" or equal-treatment principle. To voting theorists it is usually called the principle of anonymity: who you are should not affect your influence on the election. But rank order voting and majority voting both obviously satisfy anonymity in addition to the consensus principle. And so they remain to be distinguished.

A third principle often regarded as important for democratic societies is the idea that all candidates should compete on an equal footing, that the rules ought not be biased against or in favor of any one of them (it should not be the case, for example, that Buchanan requires a two-thirds majority while everyone else requires a simple majority to be elected). By analogy with equal treatment of voters, this principle could be called "equal treatment of candidates," but in fact is termed neutrality by voting theorists. However, it is still not enough to drive a wedge between majority rule and rank order voting; once again, both methods satisfy it.

So according to what principle do these voting rules differ? Well, the easiest way to introduce the basic axiom that distinguishes them is to imagine what would happen if Buchanan dropped out. In that case we would have an election with three candidates. Clearly, from the standpoint of majority rule, nothing would change: Gore would still win. And there is a sense in which nothing should change. After all, Buchanan is deemed inferior to any other candidate by 60% of the population, and he gets the lowest point total in the rank order voting calculus. So, such a candidate-who stands no chance of winning himself-ought not affect the election by his decision to run or not. Indeed, under majority rule, he doesn't affect it: if Buchanan disappears, Gore still wins handily.

But let's see what happens with rank order voting. With only three candidates, the rules dictate that a voter's favorite candidate will get 3 points, the second favorite 2 points, and the least favorite 1 point. Given the population of voters, Gore will now have 220 points (3 times 60 plus 1 times 40). Similarly, Bush gets 180 points, but Nader now only 200 points. And so the withdrawal of Buchanan means that Gore wins. In other words, rank order voting fails to satisfy the principle that "irrelevant" candidates—candidates who have no chance to win—should not determine the outcome of the election either by their absence or presence (the independence of irrelevant candidates principle).

Majority rule satisfies this principle, not just in the example but in general. You may be puzzled by this assertion because in the 2000 election there had been a lot of discussion about whether Nader's presence in the race, let alone Buchanan's, might affect the race between Bush and Gore. [In the event, it appears that Nader's presence may well have tipped the election to Bush.] But Presidential and Congressional elections are in fact not conducted according to true majority rule; in these elections, voters can express only their top choices, whereas true majority

rule typically requires information about other candidates in a voter's ranking.

More specifically, in true majority rule the winner is the candidate who beats everyone else in a pairwise comparison. But that candidate may not be identifiable just by looking at voters' top choices. To see this, it will be slightly easier to suppose for the moment that Gore, Bush, and Nader are candidates in a Senate race rather than a presidential election (Senate races are determined by plurality: the winner is simply the candidate with the most votes; there is no majority requirement). Let's imagine that the population divides into three groups. Everyone in the first group, amounting to 30% of the population, ranks Gore above Bush and Bush above Nader; in the second group—36% of the population—people rank Bush above Gore and Gore above Nader; and the third group, comprising 34% of the population, has the ranking Nader/Gore/Bush (see Table 4). If we allowed each voter to name just her favorite candidate (her top-ranked candidate), then Bush would obtain a plurality of 36% (coming from the second group), and would therefore be elected. But examining this configuration, one can see that Gore is actually the overwhelming majority favorite because 64% of the population (those in the first and third groups) prefer Gore to Bush, and 66% (those in the first and second groups) prefer Gore to Nader. So unquestionably Gore should win from the majority standpoint.

One might ask whether the true majority winner—the candidate who wins all pairwise comparisons—would always be selected by a run-off system, as used in French presidential elections, in which the two biggest vote-getters from a first round are pitted against each other in a second round. The answer unfortunately is no, as the example that we just examined illustrates. In that example, Bush and Nader would be the vote leaders in the first round: Bush with 36% and Nader with 34%. Remarkably, the majority-winner, Gore, does not even make it to the run-off!

30%	36%	34%
Gore	Bush	Nader
Bush	Gore	Gore
Nader	Nader	Bush

Table 4

3. A Problem With Majority Rule

As things stand in our discussion, majority rule appears superior to rank order voting in the sense that both of these election mechanisms satisfy the principles of consensus, anonymity (equal treatment of voters), and neutrality (equal treatment of candidates), but majority rule alone satisfies the independence of irrelevant candidates principle.

Still, there is a serious problem with majority rule, which can be understood by considering yet another hypothetical electorate. Let's suppose that a third of the population prefers Gore to Bush to Nader, a third prefers Bush to Nader to Gore, and a third Nader to Gore to Bush (see

Table 5). What would happen under majority rule with such an electorate? Notice first that two out of three voters prefer Gore to Bush (those in the first and third group); two out of three prefer Bush to Nader (those in the first and second group). But two out of three prefer Nader to Gore (those in the second and third group). In other words, no matter which candidate is chosen, two out of three people prefer somebody else. And so there's a sense in which, given this kind of electorate, there's no candidate who is the appropriate winner.

33%	33%	33%
Gore	Bush	Nader
Bush	Nader	Gore
Nader	Gore	Bush

Table 5

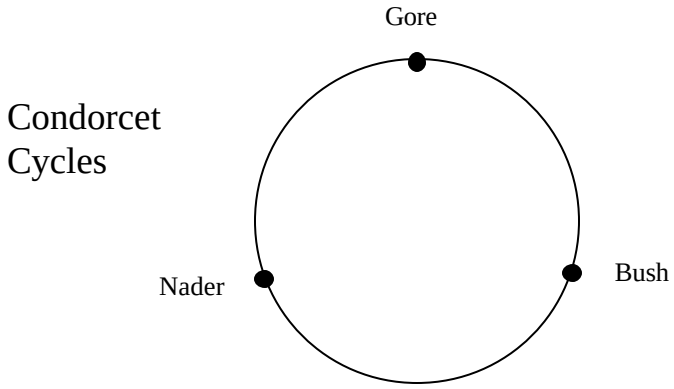
This possibility, called the Condorcet Paradox (or the Paradox of Voting), was identified by the Marquis de Condorcet, who happens to have been an exact contemporary colleague and arch-enemy of Jean-Charles Borda, whom I mentioned earlier. Both Condorcet and Borda were members of the French Academy of Sciences, and interestingly their work on voting was inspired by the need to devise a suitable voting rule for election to the Academy. Their papers on the subject inspired an enormous literature, which continues to this day. One nineteenth-century figure who was inspired to join the fray was the Oxford mathematics don Charles Dodgson, better known as Lewis Carroll.

The three preference rankings in Table 5 constitute a Condorcet cycle (the reason for the term "cycle" is that if we place Gore, Bush, and Nader on a circle—see Table 6—these three rankings are generated by moving around the circle clockwise starting with each of the three candidates in turn). Condorcet cycles are significant because, as we have seen, if voters' preferences are drawn from such a cycle, then majority rule may not produce a winner.

Technically, we then say that majority rule is intransitive. To understand this concept, consider an individual voter. If she prefers Gore to Bush and Bush to Nader, then it stands to reason she ought to prefer Gore to Nader. This seems a necessary property of logically coherent preferences, and is called transitivity by choice theorists (its violation is called intransitivity). Now, an election mechanism implicitly gives rise to what can be thought of as a social preference ranking. Thus, if majority rule is the mechanism, then A can be viewed as "socially ranked" above B if a majority prefer A to B. The mechanism is transitive if whenever A is ranked above B and B is ranked above C, then A is ranked above C. But the reason for imposing transitivity on social preferences is not so much a matter of logical coherence as to ensure that there is an electoral winner. When transitivity fails, as it does in the Condorcet Paradox, it may not be clear what the outcome of the election is.

Condorcet cycles are intimately related to the intransitivity of majority rule. Not only can the social ranking corresponding to majority voting be intransitive when the electorate's rankings

include a Condorcet cycle (as the Condorcet Paradox exemplifies), but it can be shown, in fact, that only in this case is the majority ranking intransitive.



Gore	Bush	Nader
Bush	Nader	Gore
Nader	Gore	Bush

Table 6

Now, let's go back to rank order voting. Transitivity is a principle for which rank order voting has the edge over majority rule; by its very definition, rank order voting will always generate a transitive social ranking: if Gore gets more points than Bush, and Bush gets more than Nader, then Gore obviously gets more than Nader.

So, to summarize, our comparison of majority rule and rank order voting appears to have resulted in a dead heat: majority rule satisfies all but one principle (transitivity); rank order voting also satisfies all but one principle (independence of irrelevant candidates). This might lead us to ask whether we should look at alternative voting rules.

That is, we might enquire whether there is some other voting procedure that actually satisfies all these principles. The answer, unfortunately, is "no." That answer was provided by the economist Kenneth Arrow in what is now called the Arrow Impossibility Theorem (Arrow [1951]). Arrow showed that, even leaving aside the equal treatment principles (anonymity and neutrality), any election rule that satisfies the consensus, independence, and transitivity principles must be a dictatorship. That is, the election rule must have the property that there is a particular voter whose ranking is always exactly mirrored by the social ranking. In a dictatorial election mechanism, only one voter's ranking is given any weight at all; the others' are entirely ignored. Thus, a dictatorship violates anonymity, and so Arrow's theorem implies that no voting rule satisfies all our principles.

4. Reasonable Election Rules

But there is an important sense in which Arrow's theorem conveys too negative a message. The theorem supposes that for an election rule to satisfy a given principle, it has to satisfy that principle regardless of what voters' preference rankings turn out to be. Yet some rankings might be

quite unlikely. Let me suggest that, in particular, a Condorcet cycle might well be implausible. To see my point, note that a voter's preference ranking normally does not come out of thin air. Quite possibly it derives from his or her ideological perspective. Let's consider the ideological continuum ranging from the left to the right, and let us locate each candidate on this spectrum. Then, moving from left to right, the order will presumably be Nader, Gore, Bush, and Buchanan (we're leaving aside the issue of the ideological distance between candidates). If ideology is driving voters' views, then any voter who prefers Nader to Gore is also going to prefer Gore to Bush. Similarly, any voter who prefers Bush to Gore is going to prefer Gore to Nader. And in particular we cannot have the ranking Bush/Nader/Gore, or at least that ranking seems pretty improbable if ideology is important. But notice that Bush/Nader/Gore was an important ingredient in generating the Condorcet cycle in our earlier example. In fact, it's not difficult to show that if preference rankings are ideologically driven, Condorcet cycles cannot occur. This was discovered by the political scientist Duncan Black in the late 1940's (Black [1948]). That discovery made a great deal of theoretical work in political science possible because, by assuming ideological preferences, researchers could circumvent the Condorcet Paradox and make well-defined predictions about the outcome of majority rule.

All this is to say that, in comparing voting rules, we should take account of the fact that not all preference rankings are plausible or probable. Perhaps rankings are restricted for ideological reasons, perhaps they are restricted for other reasons. But one way or another it is likely that they are going to be restricted. This brings me to some work that I've done with Partha Dasgupta of Cambridge University (Dasgupta and Maskin [1999]). We're specifically interested in comparing voting rules under the assumption that individual voters' rankings are not arbitrary but restricted to certain classes.

To see how we go about this, let me first introduce the term "reasonable" election rule. Let's call an election rule reasonable if it satisfies the principles that we've been talking about: consensus, anonymity, neutrality, independence of irrelevant candidates, and transitivity. We know from the Arrow theorem that no election rule is reasonable when voters' rankings are completely unrestricted. So consider voting rules that are reasonable for restricted classes of rankings. We'll call a rule reasonable for a class of rankings if it satisfies our four principles when voters' rankings are limited to that class. So, for example, majority rule is reasonable when rankings are restricted to those that are ideologically driven.

Our main conclusion, which takes the form of a theorem, is that majority rule is reasonable for a bigger collection of classes of rankings than any other election method. Let me make this more precise. Let's consider some other voting rule—rank order voting or plurality rule, for example—and identify a class of voters' rankings for which the election rule in question is reasonable (so that the rule satisfies all our principles when rankings belong to this class). We show that majority rule will also be reasonable for that class. Furthermore, we can find some other class of rankings for which majority rule is reasonable and this other voting rule is not.

So there is a clear sense in which majority rule dominates any other possible election rule from the standpoint of the principles that we've enunciated. Whenever some voting rule works well in the sense of satisfying these principles, majority rule does too, and there must exist cases where majority rule works well (satisfies the principles), but the other voting rule does not. We say, therefore, that majority rule is more robust than other rules.

Let me illustrate this conclusion by comparing majority rule to rank order voting. Suppose that each voter in society has either the ranking Gore/Nader/Bush or the ranking Nader/Gore/Bush. Now, with three candidates, there are six logically possible rankings. So by restricting our class to only two rankings, we have eliminated four rankings. It is not hard to see that rank order voting satisfies independence of irrelevant candidates on this class, e.g., whether

Nader or Gore wins a rank order election will not be affected by whether Bush is running or not when every voter has ranking Gore/Nader/Bush or Nader/Gore/Bush. So rank order voting is reasonable for this class (the only principle that is ever potentially problematic for rank order voting is independence). But notice that majority rule is reasonable for this class too because, as we noted earlier, majority rule fails to satisfy transitivity (the only principle that is potentially problematic for it) only when voters' preferences include a Condorcet cycle of three rankings, and here there are only two preference orderings.

Now, let us add a third ranking, Bush/Gore/Nader, to the class. Majority rule is reasonable for this expanded class too, because these three rankings do not constitute a Condorcet cycle (recall that Condorcet cycles are generated by moving around a circle of candidates starting at different points and notice that the rankings Gore/Nader/Bush, Nader/Gore/Bush, Bush/Gore/Nader cannot be generated in such a fashion). But it's not hard to check that rank order voting is not reasonable for this class. If we assign 25% of the population to the ranking Gore/Nader/Bush, 51% to ranking Bush/Gore/Nader, and 24% to ranking Nader/Gore/Bush, it will matter whether or not Nader is running. If Nader runs and there is a population of 100, you can calculate that Gore wins with a point total of 225 to Bush's 202. But if Nader drops out, it's a straight race between Gore and Bush and so, since 51% of the population prefer Bush to Gore, Bush wins. We have therefore identified a class of preferences for which rank order voting is not reasonable but majority rule is.

What is the moral of this story? One possible lesson comes from the observation that majority rule is used by virtually every democratic legislature in the world. It may be no accident that the theoretically most robust voting method is also the method with the greatest popularity. But even if one ignores that connection, I think it is still notable from a normative point of view that there is a precise sense in which majority rule does a better job than any other electoral rule in representing voters' views. So perhaps the next time the Senate passes some absurd bill, you can take consolation from the notion that although they may not have chosen correctly, they at least used the correct method for choosing.

REFERENCES

Arrow, K. [1951], *Social Choice and Individual Values*, Monograph 12, John Wiley & Sons, New York.

Black, D. [1948], "On the Rationale of Group Decision-Making," *Journal of Political Economy*, Vol. 56: 23-34.

Dasgupta, P. and E. Maskin [1999], "On the Robustness of Majority Rule," mimeo.